Issued: April 2, 2024 ISE/OR/MA 766 HW #4 Due: April 11, 2024

Exercises:

Problem1: (20 pts)

Proved that the Gale-Shapley's man-optimal algorithm (page 212, Theorem 10.1 of Lawler's book) takes no more that $n^2 - 2n + 2$ stages/steps to reach a final solution (line 8 of page 213).

Problem 2: (35 pts)

A factory expects to work on two shifts to finish nine jobs, namely, Job 1, Job 2, .., Job 9. The workers Abe, Barbara, Cindy, David, and Ellen are willing to work on the first shift, while Fox, George, Harvey, Issac, and Jean are willing to work on the second shift. Their working capability are given below:

1. Abe can finish Jobs 1 and 2 in the first shift,
2. Barbara can finish Jobs 3, 4 and 5 in the first shift,
3. Cindy can finish Job 6 in the first shift,
4. David can finish Job 7 in the first shift,
5. Ellen can finish Jobs 8 and 9 in the first shift,
6. Fox can finish Jobs 1 and 3 in the second shift,
7. George can finish Job 8 in the second shift,
8. Harry can finish Job 2 in the second shift,
9. Issac can finish Jobs 4, 6 and 7 in the second shift,
10. Jean can finish Jobs 5 and 9 in the second shift.

To minimize the total wages, the factory manager would like to use a minimum number of workers to finish all jobs in one day (with two shifts). You are asked to help the manager by taking the following steps:
(a) (10 pts) Formulate this task as a matching problem in explicit form.
(b) (5 pts) Write the dual problem of (a).
(c) (5 pts) Provide the optimality conditions of the matching problem.
(d) (10 pts) Use a matching algorithm you learned in this semester to provide an optimal solution. Remember to do it step by step.
(e) (5 pts) Tell the manager explicitly who should be hired to work on which job in which shift.

Problem 3: (45 pts)

A machine shop has six different drilling machines. On a given day, five jobs that need drilling arrive. The number of person-hours required to perform each job on each of the machines is given below.

Job / Machine	A	B	C	D	E
1	5	7	6	4	9
2	8	10	3	4	7
3	6	11	5	4	7
4	5	8	7	3	9
5	3	6	4	2	7
6	3	7	5	3	7

You are asked to find the best way to assign each job to a different machine such that the total processing time is minimized.
(a) (10 pts) Formulate this task as a matching problem in explicit form.
(b) (5 pts) Write the dual problem of (a).
(c) (5 pts) Provide the optimality conditions of the matching problem.
(d) (10 pts) Use a matching algorithm you learned in this semester to provide an optimal solution. Remember to do it step by step!
(e) (5 pts) Write down explicitly which job should be assigned to which machine.
(f) (10 pts) You are asked again to find the best way to assign each job to a different machine such that all jobs are finished at the earliest time. Please provide an optimal solution by using a matching algorithm you learned this semester. Remember to do it step by step!