Issued: Oct. 22, 2009 **IE/MA/OR505 HW 8** **Due: Oct. 29, 2009**

Exercises

(1) Textbook 4.18

(2) Consider the following linear programming problem:

$$\begin{array}{llllllllllll}
\text{Min} & 15x_1 & + & 64x_2 & - & 6x_3 & - & 6x_4 & - & 8x_5 & & \\
\text{s.t.} & x_1 & + & 8x_2 & - & x_3 & - & 2x_4 & - & x_5 & = & 0 \\
 & x_1 & - & x_2 & - & 3x_3 & - & x_4 & & & = & -1 \\
\end{array}$$
$$x_1 \; , \; x_2 \; , \; x_3 \; , \; x_4 \; , \; x_5 \; \geq 0$$

(a) Derive its dual linear programming problem.

(b) Solve the dual problem by using the graphic method.

(c) Write down the Complementary Slackness conditions for this LP problem.

(d) Use (c) to find an optimal solution to the primal problem.

(3) Consider the following linear programming problem:

$$\begin{array}{lllllllllll}
\text{Min} & c_0 & + & c_1x_1 & + & c_2x_2 & + & \cdots & + & c_nx_n & & \\
\text{s.t.} & & & a_{11}x_1 & + & a_{12}x_2 & + & \cdots & + & a_{1n}x_n & = & 0 \\
 & & & a_{21}x_1 & + & a_{22}x_2 & + & \cdots & + & a_{2n}x_n & = & 0 \\
 & & & \vdots & & \vdots & & & & \vdots & & \vdots \\
 & & & a_{m1}x_1 & + & a_{m2}x_2 & + & \cdots & + & a_{mn}x_n & = & 0 \\
 & & & x_1 & + & x_2 & + & \cdots & + & x_n & = & 1000 \\
\end{array}$$
$$x_j \geq 0 \; , \quad \text{for all } j$$

(a) What can you say about the optimal objective value of this linear programming problem?

(b) What can you say about the feasibility of its dual problem?

(c) Give explicit reasons to support your answer in part (b).

(4) Consider the following LP

$$\begin{array}{ll} \text{Min} & c^T\mathbf{x} \\ \text{s.t.} & A\mathbf{x} = b \\ & \mathbf{x} \geq 0 \end{array}$$

where A is an m×n matrix and b and c are vectors of proper dimensions. Assume that $\mathbf{x}^*$ is an optimal solution of LP and $\mathbf{w}^*$ be an optimal dual solution.

(a) Now replace c by a new cost vector $\bar{c}$ and assume that $\bar{\mathbf{x}}$ is an optimal solution to the new LP with $\bar{c}$ as the objective function. Show that $(\bar{c} - c)^T(\bar{\mathbf{x}} - \mathbf{x}^*) \leq 0$.

(b) Let the cost vector be fixed at c, but change b to a new right-hand-side vector b^0 and assume that $\mathbf{x^0}$ is a corresponding optimal solution to the new LP. Prove that $(b^0 - b)^T\mathbf{w}^* \leq c^T(\mathbf{x^0} - \mathbf{x}^*)$.

(c) What are the meanings of the results (a) and (b)?