

## Exercises

Textbook Chapter 2

1. 2.7
2. 2.9
3. 2.10
4. 2.11
5. Prove that the first orthant of  $R^n$ , i.e., the set  $R_+^n = \{\mathbf{x} \in R^n \mid x_j \geq 0, \forall j = 1, 2, \dots, n\}$ , is a convex cone that contains all of its boundary points. (We say it is a closed convex cone.)
6. In  $R^2$ , define the set  $K_2 = \{\mathbf{x} \in R^2 \mid x_1 \geq |x_2|\}$ .
  - (1) Draw the picture of  $K_2$ .
  - (2) Prove that  $K_2$  is a closed convex cone.
  - (3) What's the relationship between  $R_+^2$  and  $K_2$ ?
7. In  $R^3$ , define the set  $K_3 = \{\mathbf{x} \in R^3 \mid x_1 \geq \sqrt{x_2^2 + x_3^2}\}$ .
  - (1) Draw the picture of  $K_3$ .
  - (2) Prove that  $K_3$  is a closed convex cone.
  - (3) What's the fundamental difference between  $R_+^3$  and  $K_3$ ?
8.  $K_2$  and  $K_3$  are called Lorentz Cone in  $R^2$  and  $R^3$ , respectively.
  - (1) Please define the Lorentz cone  $K_n$  in  $R^n$  in a similar way.
  - (2) Please prove that  $K_n$  is a closed convex cone.