

## Exercises

- (1) Consider the problem

$$\begin{aligned} \text{minimize} \quad & z = 2x_1 + x_2 \\ \text{subject to} \quad & x_1 + 2x_2 = 4 \\ & x_1, x_2 \geq 0. \end{aligned}$$

(a) Plot the feasible region. Suppose the current point is  $x = (2, 1)^T$ . Define the corresponding affine scaling mapping, find the transformed linear program, and plot its feasible region.

(b) Do the same as in (a) for  $x = (3, \frac{1}{2})^T$ .

- (2) Use the primal affine scaling method to solve the problem

$$\begin{aligned} \text{minimize} \quad & z = 2x_1 + x_2 + 3x_3 \\ \text{subject to} \quad & x_1 + 2x_2 - x_3 = 1 \\ & x_1 + x_2 + x_3 = 1 \\ & x_1, x_2, x_3 \geq 0. \end{aligned}$$

Start from the point  $x_0 = (0.25, 0.5, 0.25)^T$ . At each iteration use a step length that is 0.99 of the maximum step to the boundary. Terminate when the relative improvement in the objective is below  $10^{-5}$ .

- (3) (for PhD Students) Prove that the projected steepest-descent direction for the primal problem is the solution to

$$\begin{aligned} \text{minimize}_{\quad p} \quad & \|p + c\| \\ \text{subject to} \quad & Ap = 0. \end{aligned}$$